2015-DSE
MATH CP
PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2015

Please stick the barcode label here.

Candidate Number									

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

8.30 am – 10.45 am (2¼ hours)
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

2. This paper consists of THREE sections, A(1), A(2) and B.

3. Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

©香港考試及評核局 保留版權
Hong Kong Examinations and Assessment Authority
All Rights Reserved 2015

* A 0 3 0 E 0 0 1 *

SECTION A(1) (35 marks)

1. Simplify $\dfrac{m^9}{(m^3n^{-7})^5}$ and express your answer with positive indices. (3 marks)

2. Make b the subject of the formula $\dfrac{4a+5b-7}{b}=8$. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

3. Bag A contains four cards numbered 1, 3, 5 and 7 respectively while bag B contains five cards numbered 2, 4, 6, 8 and 10 respectively. If one card is randomly drawn from each bag, find the probability that the sum of the two numbers drawn is less than 9. (3 marks)

4. Factorize

(a) $x^3 + x^2y - 7x^2$,

(b) $x^3 + x^2y - 7x^2 - x - y + 7$.

(4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

5. (a) Find the range of values of x which satisfy both $\frac{7-3x}{5} \leq 2(x+2)$ and $4x-13>0$.

(b) Write down the least integer which satisfies both inequalities in (a).

(4 marks)

6. The cost of a book is $\$250$. The book is now sold and the percentage profit is 20%.

(a) Find the selling price of the book.

(b) If the book is sold at a discount of 25% on its marked price, find the marked price of the book.

(4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

7. The number of apples owned by Ada is 4 times that owned by Billy. If Ada gives 12 of her apples to Billy, they will have the same number of apples. Find the total number of apples owned by Ada and Billy. (4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

8. In Figure 1, $ABCD$ is a circle. E is a point lying on AC such that $BC = CE$. It is given that $AB = AD$, $\angle ADB = 58°$ and $\angle CBD = 25°$.

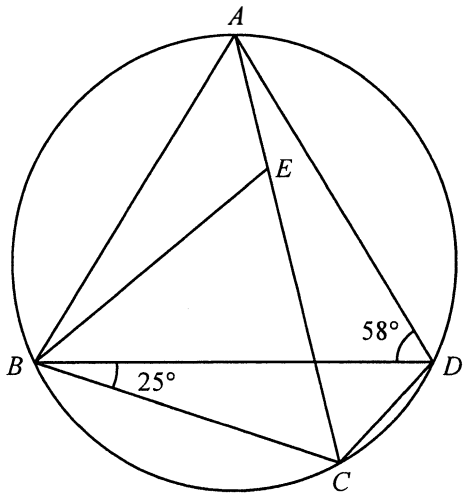

Figure 1

Find $\angle BDC$ and $\angle ABE$. (5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

9. The radius and the area of a sector are 12 cm and 30π cm^2 respectively.

(a) Find the angle of the sector.

(b) Express the perimeter of the sector in terms of π.

(5 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

SECTION A(2) (35 marks)

10. When Susan sells n handbags in a month, her income in that month is $\$S$. It is given that S is a sum of two parts, one part is a constant and the other part varies as n. When $n = 10$, $S = 10\,600$; when $n = 6$, $S = 9\,000$.

(a) When Susan sells 20 handbags in a month, find her income in that month. (4 marks)

(b) Is it possible that when Susan sells a certain number of handbags in a month, her income in that month is \$18 000 ? Explain your answer. (2 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

11. Let $f(x) = (x-2)^2(x+h)+k$, where h and k are constants. When $f(x)$ is divided by $x-2$, the remainder is -5. It is given that $f(x)$ is divisible by $x-3$.

(a) Find h and k. (3 marks)

(b) Someone claims that all the roots of the equation $f(x) = 0$ are integers. Do you agree? Explain your answer. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

12. The stem-and-leaf diagram below shows the distribution of the weights (in kg) of the students in a football club.

Stem (tens)	Leaf (units)						
4	0	2	3	3	3	3	9
5	1	1	2	2	3	7	9
6	3	5	8	9			
7	8	9					

(a) Find the mean, the median and the range of the above distribution. (3 marks)

(b) Two more students now join the club. It is found that both the mean and the range of the distribution of the weights are increased by 1 kg . Find the weight of each of these two students. (4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Please stick the barcode label here.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

13. In Figure 2, $ABCD$ is a square. E and F are points lying on BC and CD respectively such that $AE = BF$. AE and BF intersect at G.

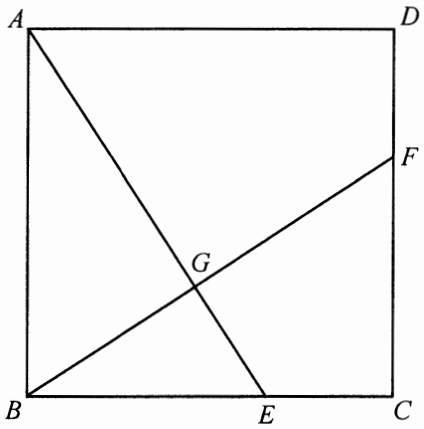

Figure 2

(a) Prove that $\Delta ABE \cong \Delta BCF$. (2 marks)

(b) Is ΔBGE a right-angled triangle? Explain your answer. (3 marks)

(c) If $CF = 15$ cm and $EG = 9$ cm, find BG. (2 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

14. The coordinates of the points P and Q are $(4, -1)$ and $(-14, 23)$ respectively.

(a) Let L be the perpendicular bisector of PQ.

(i) Find the equation of L.

(ii) Suppose that G is a point lying on L. Denote the x-coordinate of G by h. Let C be the circle which is centred at G and passes through P and Q.
Prove that the equation of C is $2x^2 + 2y^2 - 4hx - (3h + 59)y + 13h - 93 = 0$.

(6 marks)

(b) The coordinates of the point R are $(26, 43)$. Using (a)(ii), or otherwise, find the diameter of the circle which passes through P, Q and R. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

SECTION B (35 marks)

15. The table below shows the means and the standard deviations of the scores of a large group of students in a Mathematics examination and a Science examination:

Examination	Mean	Standard deviation
Mathematics	66 marks	12 marks
Science	52 marks	10 marks

The standard score of David in the Mathematics examination is -0.5 .

(a) Find the score of David in the Mathematics examination. (2 marks)

(b) Assume that the scores in each of the above examinations are normally distributed. David gets 49 marks in the Science examination. He claims that relative to other students, he performs better in the Science examination than in the Mathematics examination. Is the claim correct? Explain your answer. (2 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

16. A box contains 5 red bowls, 6 yellow bowls and 3 white bowls. If 4 bowls are randomly drawn from the box at the same time,

(a) find the probability that exactly 2 red bowls are drawn; (2 marks)

(b) find the probability that at least 2 red bowls are drawn. (2 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

17. For any positive integer n, let $A(n) = 4n - 5$ and $B(n) = 10^{4n-5}$.

(a) Express $A(1) + A(2) + A(3) + \cdots + A(n)$ in terms of n. (2 marks)

(b) Find the greatest value of n such that $\log(B(1)B(2)B(3)\cdots B(n)) \leq 8\,000$. (3 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

18. Let $f(x) = 2x^2 - 4kx + 3k^2 + 5$, where k is a real constant.

(a) Does the graph of $y = f(x)$ cut the x-axis? Explain your answer. (2 marks)

(b) Using the method of completing the square, express, in terms of k, the coordinates of the vertex of the graph of $y = f(x)$. (3 marks)

(c) In the same rectangular coordinate system, let S and T be moving points on the graph of $y = f(x)$ and the graph of $y = 2 - f(x)$ respectively. Denote the origin by O. Someone claims that when S and T are nearest to each other, the circumcentre of ΔOST lies on the x-axis. Is the claim correct? Explain your answer. (4 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

19. In Figure 3(a), $ABCDB'$ is a pentagonal paper card. It is given that $AB = AB' = 40$ cm, $BC = B'D = 24$ cm and $\angle ABC = \angle AB'D = 80°$.

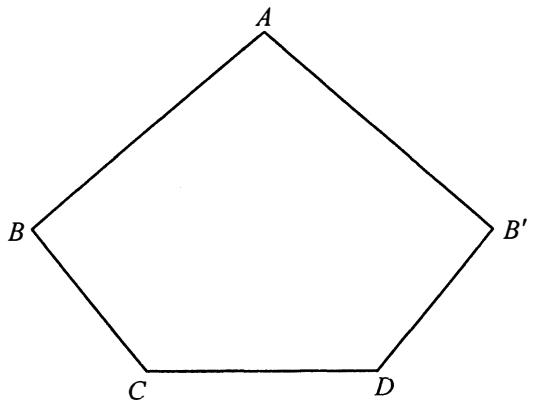

Figure 3(a)

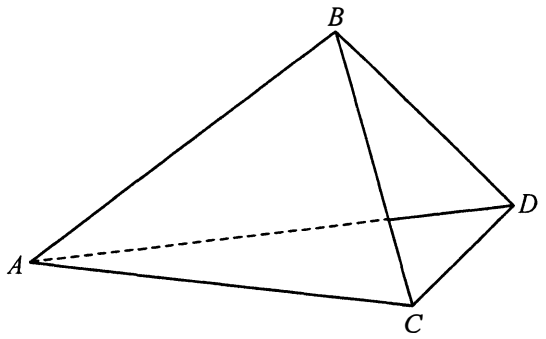

Figure 3(b)

(a) Suppose that $105° \leq \angle BCD \leq 145°$.

(i) Find the distance between A and C.

(ii) Find $\angle ACB$.

(iii) Describe how the area of the paper card varies when $\angle BCD$ increases from $105°$ to $145°$. Explain your answer.

(7 marks)

(b) Suppose that $\angle BCD = 132°$. The paper card in Figure 3(a) is folded along AC and AD such that AB and AB' join together to form a pyramid $ABCD$ as shown in Figure 3(b). Find the volume of the pyramid $ABCD$. (6 marks)

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

Go on to the next page

Answers written in the margins will not be marked.

Answers written in the margins will not be marked.

END OF PAPER

Answers written in the margins will not be marked.